## Direct Current Circuits (E4)

## Objectives

- Measure the emf and internal resistance for a battery.
- Use Kirchhoff's rules to solve simple circuit problems.


## Theory

In the previous experiment, you learned about current, voltage, and resistance. Now, you will use this knowledge to construct and examine various direct current (DC) circuits. Before you begin this experiment, you should review some of the basic concepts associated with DC circuits.

## A. Direct vs. Alternating Currents

The current in a wire (or electrical component) is just the flow of charge from one end of a device to the other. Negative charges move in one direction, which is equivalent to positive charges going the other direction. The actual direction of the current is defined as the direction in which the positive charges flow (i.e., the average velocity of charges). In metals, current is curried by negative changes (electrons), so the direction of current is opposite to the average velocity of electrons. The rate of this electrical flow is measured in amperes (A), or amps for short. One amp is equal to the flow of one coulomb of charge per second. The common equation given for current is:

$$
I=\frac{\Delta q}{\Delta t}, \text { where } I \text { is current, } q \text { is charge and } t \text { is time. }
$$

There are two basic types of current: direct (DC) and alternating (AC). The DC current flows in one direction, whereas the AC current changes its direction periodically. An example of DC current versus time graph is shown below.


In an alternating current, the charges move first in one direction and then in the opposite.
Direct currents are usually created by sources such as batteries or DC power supplies. These sources can produce such current because they always keep one terminal at a high potential and
the other at a low potential. For example, the positive end of a battery is always at the high potential, while the negative end remains at a low one. This polarity does not switch.

## B. Electromotive Force

The strength of potential energy sources (a battery in our case) is measured by a quantity called electromotive force $\boldsymbol{\mathcal { E }}$. Specifically, the electromotive force (emf) is the amount of electric energy delivered by the source per coulomb of charge as this charge passes through the source from the low-potential terminal to the high-potential terminal. Because the emf is measured in energy per unit charge, it is not actually a force. Instead, it is the voltage of the source, measured in volts.

Drops in potential $(V)$ occur as the current encounters any resistance within the circuit. In simple circuits, this resistance mainly comes from resistors $\mathrm{R}_{1}, \mathrm{R}_{2}$, etc., within the circuit and from the battery itself that has an internal resistance $r$.

According to Ohm's Law, the potential drop across any resistor is given by: $V=I R$. Similarly, the potential drop across the battery is $V_{\text {battery }}=\mathcal{E}-I r$, where $r$ is the internal resistance of the battery. Each of these reductions in potential must be compensated for by the emf of the battery.


$$
\begin{gather*}
\mathcal{E}-I r-I R=0 \Rightarrow \mathcal{E}=I(r+R)=I r+I R=I r+V  \tag{1}\\
\text { or } \quad V=\mathcal{E}-I r \\
\mathcal{E}-I r-I R=0 \Rightarrow \text { if } R \gg r, \text { then } I R \gg I r \text { and } I r \text { could be ignored }  \tag{2}\\
\text { In that case: } \mathcal{E} \cong I R=V
\end{gather*}
$$

## C. Kirchhoff's Rules

Kirchhoff's junction rule says that the sum of the currents entering any junction point must be equal to the sum of the currents leaving that junction.

Kirchhoff's loop rule states that around any closed loop in a circuit, the sum of all the electromotive forces and all the potential changes measured across resistors and other circuit elements must equal zero.

The best way to understand this is through an example. The circuit shown below contains an emf source and two resistors. The emf plus the total voltage for this circuit must equal zero.


$$
\begin{aligned}
& \varepsilon+\Delta V=0 \\
& \mathcal{E}-I r_{\text {battery }}-I R_{1}-I R_{2}=0 \\
& \varepsilon=I r_{\text {battery }}+I R_{1}+I R_{2} \\
& \varepsilon=I r_{\text {battery }}+V_{1}+V_{2} \\
& \mathcal{E}=I r_{\text {battery }}+V_{T} \\
& V_{T}=\varepsilon-I r_{\text {battery }}
\end{aligned}
$$

This process becomes more complicated if there is more than one loop present in the circuit. To find the currents in such a system, one needs to solve the set of equations made from the single loops. These equations may be determined from a process called the loop method:

1. Divide the circuit into individual closed current loops. These loops may partially overlap.
2. Assign a direction to each of the currents in the loop and label them $I_{1}, I_{2}, I_{3}$. This direction is arbitrary, and if it is incorrect, the value of the current will turn out to be negative. Apply Kirchhoff's first rule to the junction point(s).
3. Apply Kirchhoff's second rule to each individual loop. When calculating the potential change across a resistor, you must take the product of the resistance and the net current through the resistor.

Again, this process is best understood through an example. The circuit below may be split into two separate loops with currents $\mathrm{I}_{1}, \mathrm{I}_{2}$, and $\mathrm{I}_{3}$. By applying both Kirchhoff's rules, you will obtain the equations necessary to find these currents.


The first Kirchhoff's rule applied to junction B gives the first equation:
(a) $I_{1}=I_{2}+I_{3}$
Note that junction E gives the same equation!

The second Kirchhoff's rule applied to the upper loop BEDC gives the second equation:
(b) $\quad \mathcal{E}=I_{2} R_{1}+I_{2} R_{2}+I_{1} r$

The second Kirchhoff's rule applied to the loop AFEB gives the third equation:
(c) $0=I_{3} R_{3}-I_{2} R_{1}-I_{2} R_{2}$

There is a third possible loop (AFDC), but use of it will only lead to redundancy. To solve for three unknowns ( $I_{1}, I_{2}, I_{3}$ ), we only need three equations - junction rule and two loops.

If you know the emf $\mathcal{E}$ of the battery, the resistance values for $R_{1}, R_{2}$, and $R_{3}$, and the battery's internal resistance $r$, then you can find $\mathrm{I}_{1}, \mathrm{I}_{2}$, and $\mathrm{I}_{3}$ by simultaneously solving the three equations (a) - (c).

$$
I_{2}=\frac{\varepsilon R_{3}}{\left[R_{1} R_{3}+R_{2} R_{3}+r\left(R_{1}+R_{2}+R_{3}\right)\right]} \quad, \quad I_{3}=I_{2} \frac{R_{1}+R_{2}}{R_{3}}, \quad I_{1}=I_{2}+I_{3}
$$

## Procedure:

## Activity 1: Measurements of Voltage, Current and Resistance Using a Digital Multimeter

In this activity, you will use the digital multimeter to measure the voltage from a type "C" battery, measure the minimum and maximum resistance of the adjustable resistor (rheostat) and set a circuit with a given value of the current.
1.1. Set the multimeter to the 2 -Volt DC range $(2 \mathrm{~V}=)$. Connecting cables should be plugged into two left holes as shown below.

$\mathrm{V}=$ d.c. voltage
$\mathrm{V} \sim$ a.c. voltage
$\mathrm{A}=$ d.c. current
$\mathrm{A} \sim$ a.c. current
$\boldsymbol{\Omega}$ resistance
1.2. Connect the multimeter prongs to the tips of the battery holder (red tip to positive side of the battery and black tip to negative side of the battery). Record this voltage on your data sheet as the $\operatorname{emf}(\mathcal{E})$ of the battery.

The potential difference that exists across the battery itself (no other devices connected) is called the $\operatorname{emf}(\boldsymbol{E})$ of the battery. With analog meters, it is difficult to measure the emf due to the error introduced by the meter itself. However, most of the digital multimeters have a very high input resistance ( $\mathrm{R}_{\text {input }}>10^{7} \Omega$ in this case $)$, which makes this measurement possible. The internal resistance of the type "C" battery $(r)$ is usually $\sim 1 \Omega$ (we will measure it in the next Activity). It is seven orders of magnitude smaller than the input resistance of the digital meter ( $\mathrm{R}_{\text {input }}$ ). From Equation (2), you
should be able to see that if $R$ is very large, then the measured voltage is very close to the emf force for the battery: $V \cong \mathcal{E}$.
1.3. Switch the multimeter to measurements of resistance, connect it to the adjustable rheostat and measure both the minimum resistance of the rheostat and the maximum resistance of the rheostat.

Set the range to $2 \mathrm{~V} \mathrm{DC}(2 \mathrm{~V}=)$ unless your multimeter is on the auto range mode. Record these values on the data sheets.

1.4. Next step is to build a circuit with a specific value of electric current. You may use any resistor that is available on the table.

Using equipment that is available on the table, build a circuit that would have a given value of DC current $I=22 \pm 1 \mathrm{~mA}$. First, calculate the total resistance needed to create current $I=22 \pm 1 \mathrm{~mA}$ for the measured value of the emf force $(\mathcal{E})$ in a single loop circuit. Connect the multimeter to measure that current.

On the data sheets, draw the circuit, record the exact current value and ask TA to verify current measurements.

## Activity 2: The Internal Resistance of type "C" Battery

In this activity, you will measure the internal resistance $r$ of type " C " battery used in Activity 1. You need to modify previously used circuits in the following steps.
2.1. First, you should attach two multimeters - one across the battery and one between the 10 $\Omega$ resistor and the rheostat.

Reminder:
The multimeter set to measure current (i.e., used as an ammeter) should be connected in series with other resistors.

The multimeter set to measure voltage (i.e., used as a voltmeter) should be connected in parallel with other resistors.

2.2. Make sure that the multimeter connected across the battery is set to the 2 -Volt DC range ( $2 \mathrm{~V}=$ ) and the one measuring current is set to the 200 mADC range $(\mathrm{A}=$ ). The multimeter connected across the battery should have wires connected to terminals labeled "COM" and "V". The multimeter measuring current should have wires connected to terminals labeled
"COM" and " 200 mA ". The wire between the knife switch and the rheostat must be connected to the slide terminal on the top of the rheostat!

Change the location of the rheostat's sliding contact to eight random locations. Use the whole length of the rheostat! At each location, record the voltage and current values found on both multimeters.

2.3. Create a voltage vs. current graph (voltage on the vertical axis and current on the horizontal axis) and attach the graph to your lab report. Calculate the straight line fit and find the slope and the $y$-intercept for that line. Find the internal resistance of the battery $r$.

## Activity 3: A Multiloop Circuit

In this activity, you will measure the currents that flow through a multiloop circuit. You will then compare these values to those predicted by Kirchhoff's rules. In your calculation, include the internal resistance of the battery $r$. Before you begin this analysis, you should disassemble your previous circuit and set up the multiloop circuit as shown below.
3.1. Make sure that the multimeter is set on the DC $\mathbf{2 0 0 m}(\mathbf{A}=)$ setting and that its leads are plugged into the "COM" and "200mA" positions.

3.2. You will first measure the current that flows through the wire connecting $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$. That current, according to Kirchhoff's rules is equal to: $\mathrm{I}_{2}$. Insert the multimeter into the circuit as shown below.

Once your circuit has been made, you may continue Activity 3 by following the steps on the following page. Please do not forget to remove the connection between $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ !


3.3. The current that is displayed is equal to $\mathrm{I}_{2}$. Record the value of $\mathrm{I}_{2}$ on your data sheets. This current is very small!
3.4. Measure the current through loop \#2 $\left(\mathrm{I}_{3}\right)$ by inserting the multimeter into the connection between $R_{2}$ and $R_{3}$ as shown in the following picture.
knife switch -
(should be closed for

3.5. The current that is displayed is $\mathrm{I}_{3}$. Record this current value on your data sheet.
3.6. Disassemble all components in your circuits and turn off both multimeters.

Students are expected to complete the lab report and return it to the lab TA before the end of the scheduled lab time.

## Make sure to complete the following tasks:

You must submit the answers to the prelaboratory questions online.

1. Your completed graph for Activity 2.
(Title and write your name and those of your partners on each graph.)
2. Your completed Data Sheets.
3. Return the completed lab report to your lab TA.
